Chapter 4
Modeling of MOS Transistors Using SPICE

S.M. Kang, Y. Leblebici, and C. Kim
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1. Prob. 3.1 (textbook)
2. Prob. 3.6
3. Prob. 3.7
4. Prob. 3.10
5. Prob. 3.14
Introduction(1)

- SPICE – Simulation Program with Integrated Circuit Emphasis
- SPICE is used very widely both in the microelectronics industry and in educational institutions
- The physical aspects of various MOSFET models used in SPICE will be described in this chapter
Introduction(2)

◆ The origin SPICE (UC Berkeley 1970s) has 3 built-in MOSFET models:
  ▪ LEVEL 1 (MOS1) square-law current-voltage characteristic
  ▪ LEVEL 2 (MOS2) analytical MOSFET
  ▪ LEVEL 3 (MOS3) semi-empirical model

◆ The level of the MOSFET model is declared on the .MODEL statement

◆ Example:

```
M1 3 1 0 0 NMOD  L=1U  W=10U  AD=120P  PD=42U
MDEV32 14 9 12 5 PMOD  L=1.2U  W=20U
.MODEL NMOD NMOS  (LEVEL=1  VTO=1.4  KP=4.5E-5  CBD=5PF  CBS=2PF)
.MODEL PMOD PMOS  (VTO=-2  KP=3.0E-5  LAMBDA=0.02  GAMMA=0.4 + CBD=4PF  CBS=2PF  RD=5  RS=3  CGDO=1PF + CGSO=1PF  CGBO=1PF)
```
Basic Concepts(1)

- **$I_D$** determines the steady-state current-voltage the device’s behavior
- **$R_D$** and **$R_S$** are the parasitic source and drain resistances

*Figure 4.1.* Equivalent circuit structure of the LEVEL 1 MOSFET
Basic Concept(2)

- **W**: Channel width
- **L**: Nominal channel length
- **$L_{eff}$**: The distance on the surface between the two diffusion regions
- **$L_D$**: Lateral diffusion coefficient
The LEVEL 1 Model Equations(1)

◆ Linear Region

\[ I_D = \frac{k'}{2} \cdot \frac{W}{L_{\text{eff}}} \cdot \left[ 2 \cdot (V_{GS} - V_T) V_{DS} - V_{DS}^2 \right] \cdot (1 + \lambda V_{DS}) \quad \text{for} \quad V_{GS} \geq V_T \]

\[ \quad \text{and} \quad V_{DS} < V_{GS} - V_T \]

◆ Saturation Region

\[ I_D = \frac{k'}{2} \cdot \frac{W}{L_{\text{eff}}} \cdot (V_{GS} - V_T)^2 \cdot (1 + \lambda \cdot V_{DS}) \quad \text{for} \quad V_{GS} \geq V_T \]

\[ \quad \text{and} \quad V_{DS} \geq V_{GS} - V_T \]

◆ where:

\[ V_T = V_{T0} + \gamma \cdot \left( \sqrt{\left| 2\phi_F \right|} + V_{SB} - \sqrt{\left| 2\phi_F \right|} \right) \]
The LEVEL 1 Model Equation(2)

- The effect channel length:
  \[ L_{\text{eff}} = L - 2 \cdot L_D \]

- Other parameters:
  \[ k' = \mu \cdot C_{ox}, \quad \text{where} \quad C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} \]
  \[ \gamma = \frac{\sqrt{2 \cdot \varepsilon_{Si} \cdot q \cdot N_A}}{C_{ox}} \]
  \[ 2\phi_F = 2 \frac{kT}{q} \cdot \ln \left( \frac{n_i}{N_A} \right) \]
The LEVEL 1 Model Equation(3)

Example simulation parameters

\[ k' = 98.2 \, \mu A/V^2 \]

\[ VT0 = 0.53 \, V \]

\[ \gamma = 0.574 \, V^{1/2} \]

\[ 2\Phi F = -1.02 \]

\[ \lambda = 0 \]

Corresponding parameters

\[ \mu n = 44.7 \, \text{cm}^2/\text{V} \cdot \text{s} \]

\[ tox = 1.6 \, \text{nm} \]

\[ NA = 4.80 \times 10^{18} \, \text{cm}^{-3} \]

\[ LD = 10 \, \text{nm} \]

\[ KP = 98.2 \, \mu \text{A} \]

\[ VTO = 0.53 \]

\[ GAMMA = 0.574 \]

\[ PHI = 1.02 \]

\[ LAMBDA = 0 \]

\[ UO = 44.7 \]

\[ TOX = 1.60 \times 10^{-9} \]

\[ NSUB = 4.80 \times 10^{18} \]

\[ LD = 1.00 \times 10^{-8} \]
The LEVEL 1 Model Equation(4)

- Drain current with VTO

*Figure 4.2.* Variation of the drain current with model parameter VTO, for the LEVEL1 model.
The LEVEL 1 Model Equation(5)

Drain current with KP

Figure 4.3. Variation of the drain current with model parameter KP, for the LEVEL1 model
The LEVEL 1 Model Equation (6)

- Drain current with TOX

*Figure 4.4.* Variation of the drain current with model parameter TOX, for the LEVEL1 model
The LEVEL 1 Model Equation (7)

- Drain current with LAMBDA

Figure 4.5. Variation of the drain current with parameter LAMBDA, for the LEVEL1 model
The LEVEL 2 Model Equation (1)

◆ Drain current in term of bulk charge

\[
I_D = \frac{k'}{(1-\lambda \cdot V_{DS})} \cdot \frac{W}{L_{\text{eff}}} \cdot \left\{ \left( V_{GS} - V_{FB} - |2\phi_F| - \frac{V_{DS}}{2} \right) \cdot V_{DS} - \frac{2}{3} \cdot \gamma \cdot \left[ \left( V_{DS} - V_{BS} + |2\phi_F| \right)^{3/2} - \left( -V_{BS} + |2\phi_F| \right)^{3/2} \right] \right\}
\]

\( V_{FB} \) denotes the flat-band voltage of the MOSFET

◆ Saturation voltage – the channel charge at the drain end becomes zero

\[
V_{DSAT} = V_{GS} - V_{FB} - |2\phi_F| + \gamma^2 \cdot \left( 1 - \sqrt{1 + \frac{2}{\gamma^2} \cdot \left( V_{GS} - V_{FB} \right) } \right)
\]
The LEVEL 2 Model Equation(2)

◆ Saturation mode current:

\[ I_D = I_{Ds} \cdot \frac{1}{(1 - \lambda \cdot V_{DS})} \]

where \( I_{Ds} \) is calculated from 4.9 using saturation condition

◆ The zero-bias threshold voltage:

\[ V_{T0} = \Phi_{GC} - \frac{q \cdot N_{SS}}{C_{ox}} + |2\phi_F| + \gamma \cdot \sqrt{2\phi_F} \]

\( \Phi_{GC} \) : gate-to-channel work function difference

\( N_{SS} \) : fixed surface charge density
The LEVEL 2 Model Equation(3)

Drain current with GAMMA

Figure 4.6. Variation of the drain current with parameter GAMMA, for the LEVEL2 model
Variation of Mobility with Electric Field: to simulate the mobility variation, we use \( k'(\text{new}) \) instead of \( k' \):

\[
k'(\text{new}) = k' \cdot \left( \frac{\varepsilon_{Si}}{\varepsilon_{ox}} \cdot \frac{t_{oc} \cdot U_c}{(V_{GS} - V_T - U_t \cdot V_{DS})} \right)^{U_e}
\]

- \( U_c \): Gate-to-channel critical field
- \( U_e \): Exponential fitting parameter
- \( U_t \): Contribution of the drain voltage to gate-to-channel (value between 0 and 0.5)
The LEVEL 2 Model Equation(5)

- **Variation of Channel Length in Saturation Mode:** Channel length in saturation mode:

\[ L'_{\text{eff}} = L_{\text{eff}} - \Delta L \]

Where,

\[ \Delta L = \sqrt{\frac{2 \cdot \varepsilon_{Si}}{q \cdot N_A}} \cdot \left[ \frac{V_{DS} - V_{DSAT}}{4} + \sqrt{1 + \left( \frac{V_{DS} - V_{DSAT}}{4} \right)^2} \right] \]

- Channel length shortening (not specified in the .MODEL)

\[ \lambda = \frac{\Delta L}{L_{\text{eff}} \cdot V_{DS}} \]
Saturation of Carrier Velocity: carriers in the channel approach maximum speed limit. In saturation mode, the inversion layer charge at the channel-end is:

\[ Q_{inv} = \frac{I_{Sat}}{W \cdot v_{max}} \quad v_{max} \text{ Maximum carrier speed} \]

Channel length shortening:

\[ \Delta L = X_D \cdot \sqrt{\left(\frac{X_D \cdot v_{max}}{2 \cdot \mu}\right)^2 + V_{DS} - V_{DSAT} - \frac{X_D^2 \cdot v_{max}}{2 \cdot \mu}} \]

where,

\[ X_D = \sqrt{\frac{2 \cdot \varepsilon_{Si}}{q \cdot N_A \cdot N_{eff}}} \]
The LEVEL 2 Model Equation (7)

◆ **Subthreshold Conduction:** Because $V_{gs} < V_T$, even when the surface is not in strong inversion, there is a channel current

◆ **Subthreshold current:** diffusion between the source and the channel

◆ Drain current in the *weak inversion region*:

$$I_D(\text{weak inversion}) = I_{on} \cdot e^{(V_{GS} - V_{on}) \left(\frac{q}{nkT}\right)}$$

$I_{on}$: current in strong inversion region

And, $$V_{on} = V_T + \frac{nkT}{q}$$ where, $$n = 1 + \frac{q \cdot N_{FS}}{C_{ox}} + \frac{C_d}{C_{ox}}$$

$N_{FS}$ The number of fast superficial states

$C_d$ The capacitance associated with the depletion region
The LEVEL 2 Model Equation(8)

Drain current

Figure 4.7. Variation of the drain current in the weak inversion region, as a function of the gate voltage and for different values of the parameter $N_FS$, in the LEVEL 2 model.
The LEVEL 3 Model Equation(1)

LEVEL 3 equations are empirical
- The accuracy improvement
- The complexity of the calculation limitation

The drain current in the linear region:

\[ I_D = \mu_s \cdot C_{ox} \cdot \frac{W}{L_{\text{eff}}} \cdot \left( V_{GS} - V_T - \frac{1 + F_B}{2} \cdot V_{DS} \right) \cdot V_{DS} \]

where

\[ F_B = \frac{\gamma \cdot F_S}{4 \cdot \sqrt{2\phi_F} + V_{SB}} + F_n \]

\( F_B \) The dependence of the bulk depletion charge of the MOSFET
The LEVEL 3 Model Equation(2)

- The surface mobility on the gate electric field:

\[
\mu_s = \frac{\mu}{1 + \theta \cdot (V_{GS} - V_T)}
\]

- The effective mobility equation:

\[
\mu_{eff} = \frac{\mu_s}{1 + \mu_s \cdot V_{DS} \cdot \frac{1}{v_{max} \cdot L_{eff}}}
\]

\( m_s \)  Surface mobility
State-of-the Art MOSFET Models

- BSIM – Berkeley Short-Channel IFGET Model or LEVEL 4 model
- BSIM becomes the most popular SPICE MOSFET models at present
- BSIM4 version is used by many companies with 0.13um CMOS fabrication processes
EKV Transistor Model

- EKV – Enz-Krummenacher-Vittoz
- EKV Model attempts to resolve the serious problems in the modeling of transistors operating at very low voltages. It uses unified view of the transistor operating regions, and avoids the use of disjoint equation in strong and weak inversion
- EKV model is accurate simulation tool for analog circuits and digital circuits operating near threshold voltage
Capacitance Model(1)

- Gate Oxide Capacitances: SPICE uses a simple gate oxide capacitance

![Figure 4.8](https://example.com/image.png)

*Figure 4.8.* Oxide capacitances as functions of the gate-to-substrate voltage, according to Ward's capacitance model

TOX, channel width W, channel length L, and lateral diffusion LD are required.
Capacitance Model(2)

- Junction Capacitances: SPICE uses simple pn-junction model to simulate the parasitic capacitances of the source and drain diffusion:

\[
C_{SB} = \frac{C_j \cdot AS}{1 - \frac{V_{BS}}{\phi_0}}^{M_j} + \frac{C_{jsw} \cdot PS}{1 - \frac{V_{BS}}{\phi_0}}^{M_{jsw}}
\]

\[
C_{DB} = \frac{C_j \cdot AD}{1 - \frac{V_{BD}}{\phi_0}}^{M_j} + \frac{C_{jsw} \cdot PD}{1 - \frac{V_{BD}}{\phi_0}}^{M_{jsw}}
\]

- \(C_j\): Zero-bias depletion capacitance per unit length at the bottom junction of the drain and source
- \(C_{jsw}\): Zero-bias depletion capacitance per unit length at the sidewall junctions
For typical sidewall doping, we have:

\[ C_{jsw} \approx \sqrt{10} \cdot C_j \cdot x_j \]

- AS and AD are the source and the drain areas
- PS and PD are the source and the drain perimeters
- \( M_j \) and \( M_{jsw} \) are the junction grading coefficients for the bottom and the sidewall junctions
- Default values:
  \[ M_j = 0.5 \]
  \[ M_{jsw} = 0.33 \]
Example 4.1(1)

◆ The top view of an n-channel MOSFET:

- NA = $4.80 \times 10^{18}$ cm$^{-3}$
- NA (sidewall) = $1.51 \times 10^{15}$ cm$^{-3}$
- ND = $4.80 \times 10^{18}$ cm$^{-3}$
- $x_j = 0.032$ μm
- $tox = 16$ Å
- $L_d = 10$ nm

◆ The zero-bias threshold voltage is measured as 0.53V
- and $k' = 98.2$ uA/V$^2$
- $\lambda = 0.08$
Example 4.1(2)

- The gate oxide capacitance per unit area:

\[ C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} = \frac{3.51 \times 10^{-13}}{1.60 \times 10^{-7}} = 2.19 \times 10^{-6} \text{ F/cm}^2 \]

- The substrate bias coefficient (GAMMA):

\[ \gamma = \sqrt{\frac{2 \cdot q \cdot N_A \cdot \varepsilon_{Si}}{C_{ox}}} = \sqrt{\frac{2 \cdot 1.6 \times 10^{-19} \cdot 4.80 \times 10^{18} \cdot 1.04 \times 10^{-12}}{2.19 \times 10^{-6}}} = 0.577 V^2 \]

- The surface inversion potential (PHI):

\[
2 \cdot \phi_F (\text{substrate}) = \left| 2 \cdot \frac{kT}{q} \ln \left( \frac{n_i}{N_A} \right) \right|
\]

\[
= \left| 2 \cdot 0.026 V \cdot \ln \left( \frac{1.45 \times 10^{10}}{4.80 \times 10^{18}} \right) \right| = 1.02 V
\]
Example 4.1(3)

We can calculate:

\[
\phi_0 = \frac{kT}{q} \cdot \ln \left( \frac{N_A \cdot N_D}{n_i^2} \right) = 0.026 \text{V} \cdot \ln \left( \frac{4.80 \times 10^{18} \cdot 4.80 \times 10^{18}}{2.10 \times 10^{20}} \right) = 1.02 \text{V}
\]

\[
C_{j0} = \sqrt{\frac{\varepsilon_{Si} \cdot q \cdot \left( \frac{N_A \cdot N_D}{N_A + N_D} \right) \cdot \frac{1}{\phi_0}}{2}}
\]

\[
= \sqrt{\frac{1.04 \times 10^{-12} \text{F/cm} \cdot 1.6 \times 10^{-19} \text{C} \cdot 4.80 \times 10^{18} + 4.80 \times 10^{18}}{2}} \cdot \frac{1}{1.02}
\]

\[
= 4.42 \times 10^{-7} \text{F/cm}^2
\]
Example 4.1(4)

And

\[
C_{j0sw} = x_j \cdot \sqrt{\frac{\varepsilon_s \cdot q}{2}} \cdot \left( \frac{N_A(sw) \cdot N_D}{N_A(sw) + N_D} \right) \cdot \frac{1}{\phi_0}
\]

\[
= 3.20 \times 10^{-6} \cdot \sqrt{\frac{1.04 \times 10^{-12} \text{ F/cm} \cdot 1.6 \times 10^{-19} \text{ C}}{2}} \cdot \left( \frac{2.99 \times 10^{15} \cdot 4.80 \times 10^{18}}{2.99 \times 10^{15} + 4.80 \times 10^{18}} \right) \cdot \frac{1}{1.02}
\]

\[
= 5.00 \times 10^{-14} \text{ F/cm}
\]

We assume \( M_j = 0.5 \) and \( M_jsw = 0.33 \). We have:

\[
C_{GSO} = C_{GDO} = C_{ox} \cdot L_D = 2.19 \times 10^{-6} \cdot 10 \times 10^{-7} = 2.19 \text{ pF/cm}
\]
Example 4.1(5)

◆ SPICE statement:

M1 6 12 4 7 NM1 W=200N L=120N LD=10N AS=0.058P PS=0.98U AD=0.1492P PD=1.7I
.MODEL NM1 NMOS ( VTO=0.53 KP=98.2U LAMBDA=0.08 GAMMA=0.577
+ PHI=1.02 PB=1.02 CJ=4.42E-3 CJSW=5.00E-10
+ CGSO=2.19E-10 CGDO=2.19E-10 MJ=0.5 MJSW=0.33 )
Comparison of the SPICE MOSFET Models

- The LEVEL 1 model: usually not very precise because of GCA and small number of fitting parameters
  - Useful for a quick and rough estimate
- The LEVEL 2 model: can be used with differing complexities
  - By adding the related parameters
  - Requires a large amount of CPU time
  - Problem with convergence
Comparison of the SPICE MOSFET Models

LEVEL 2 AND LEVEL 3 comparison:

Figure 4.9. Drain current versus drain voltage characteristics of an n-channel MOSFET calculated with the LEVEL 2 model (A) and the LEVEL 3 model (B).

The parameters common for both models are: \( V_{TO} = 0.53 \), \( XJ = 3.20E-8 \), \( LD = 1.0E-8 \).

The parameters of the LEVEL 2 model are: \( UO = 44.7 \), \( UCRIT = 2.0E7 \).

The parameters of the LEVEL 3 model are: \( UO = 47.5 \), \( THETA = 0.17 \).